Lecture Notes (Math 90): Week VI (Thursday)

Alicia Harper

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Example 1. Use the inverse function theorem to compute the derivatives of $sin^{-1}(x)$, $cos^{-1}(x)$, and $tan^{-1}(x)$.

Example 2. (Redux!) Use the inverse function theorem to compute the derivative of $f(x) = ln(x)$.

Example 3. More generally, compute the derivatives of $e^a x$ and $log_a(x)$.

Example 4. Show that $\lim_{x\to 0}(1+x)^{1/x} = e$

Example 5. How does the exponential give us a definition of exponentiation x^a for *irrational* a. Use this idea to differentiate $f(x) = x^x$.

Example 6. How does knowing the derivative of $ln(x)$ help us more generally? Use logarithmic differentiation to compute the derivative of $y = \frac{(x^2+1)(x+1)^{\frac{1}{2}}}{x-1}$

Example 7. Use logarithmic differentiation to differentiate $\sqrt[2]{\frac{(x+9)^10}{(x-2)^5}}$.

Example 8. Use the inverse function theorem to compute the derivatives of $sec^-1(x)$, $csc^-1(x)$, and $cot^-1(x)$.

Example 9. Water runs into a conical tank at the rate of 9 ft3>min. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep? Recall that, for a cone, we have $V = \frac{4}{3}\pi r^3$ where V is the volume.

Example 10. A hot air balloon rising straight up from a level field is tracked by a range finder 150 m from the liftoff point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at the rate of 0.14 rad $>$ min. How fast is the balloon rising at that moment?