Lecture Notes (Math 90): Week VII (Tuesday)

Alicia Harper

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Related Rates

1 The Story

So far, we have (almost) exclusively engaged the subject differentiation in the abstract. Aside from a brief mention of velocity here, a shout out to marginal cost over there, we have eschewed most applications in favor of developing the basic conceptual techniques.

Today, we begin to deal with the fundamental question posed at the start of the course: What is Calculus *practically* used for? Most of the problems we shall deal with today are rather simple from a purely analytic perspective, with the main challenge being there comprehension: Once you understand what the problem is asking for, the resulting calculus problem should pose little difficulty to you.

Here is general strategy by which you can approach these problems (One shamelessly stolen from the text:

2 The Strategy

1. (Function and variables) Identify the function $f(x)$ under consideration. Identify other variables.

2. (Numerics) Write down all the numerical information given to you and how it related to the function $f(x)$.

3. (Illustration) Draw a picture of the situation. This is key! (At least for me)

4. (Relationship) Figure out what the problem is telling (or asking) you about the derivative $f'(x)$. Be sure that you did not confuse the derivative with $f(x)$. Now, finally, write down an equation relating all the data according to the stated problem.

5. (Solve) Solve the resulting equation. This may require any of the techniques developed so far in this course. I usually like to apply implicit differentiation first, due to it's sheer flexibility.Evaluate the solution on any numerical information if needed.

3 The Geometric Relationships

Before we begin doing some examples, let us record some basic geometric formulas that will prove useful.

1. The volume of a sphere of radius r: $\frac{4\pi}{3}r^3$
2. The surface area of a sphere of radius r: $4\pi r^2$
3. The volume of a right circular cone of height h and radius: $\frac{1}{3}\pi r^2 h$
4. The area of the side of a right circular cone of height h and radius r: $\pi r\sqrt[2]{r^2 + h^2}$.
5. The volume of a right circular cylinder of height h and radius: $\pi r^2 h$
6. The area of the side of a right circular cylinder of height h and radius r: $2\pi rh$.

Example 1. Water runs into a conical tank at the rate of $9ft/min$. The tank stands point down and has a height of $10ft$ and a base radius of $5ft$. How fast is the water level rising when the water is $6ft$ deep?

1. Function and variables

2. Numerics

3. Illustration

4. Relationship

5. Solve

Example 2. A hot air balloon rising straight up from a level field is tracked by a range finder $150m$ from the liftoff point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at the rate of $0.14 rad/min$. How fast is the balloon rising at that moment?

1. Function and variables

2. Numerics

3. Illustration

4. Relationship

5. Solve

Example 3. A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is $0.6mi$ north of the intersection and the car is $0.8mi$ to the east, the police determine with radar that the distance between them and the car is increasing at $20mph$. If the cruiser is moving at $60mph$ at the instant of measurement, what is the speed of the car?

1. Function and variables

2. Numerics

3. Illustration

4. Relationship

5. Solve

Example 4. A jet airliner is flying at a constant altitude of $12,000 ft$ above sea level as it approaches a Pacific island. The aircraft comes within the direct line of sight of a radar station located on the island, and the radar indicates the initial angle between sea level and its line of sight to the aircraft is $30°$. How fast (in miles per hour) is the aircraft approaching the island when first detected by the radar instrument if it is turning upward (counterclockwise) at the rate of $\frac{2}{3} deg/sec$ in order to keep the aircraft within its direct line of sight?

1. Function and variables

2. Numerics

3. Illustration

4. Relationship

5. Solve

Example 5. A light shines from the top of a pole $50ft$ high. A ball is dropped from the same height from a point $30ft$ away from the light. (See accompanying figure.) How fast is the shadow of the ball moving along the ground $\frac{1}{2}sec$ later? (Assume the ball falls a distance $s = 16t^2ft$ in t seconds.)

1. Function and variables

2. Numerics

3. Illustration

4. Relationship

5. Solve

Example 6. At what rate is the angle between a clock's minute and hour hands changing at 4 o'clock in the afternoon?

1. Function and variables

2. Numerics

3. Illustration

4. Relationship

5. Solve

Example 7. A spherical iron ball 8 inches in diameter is coated with a layer of ice of uniform thickness. If the ice melts at the rate of $10 inches^3/min$, how fast is the thickness of the ice decreasing when it is 2 inches thick? How fast is the outer surface area of ice decreasing?

1. Function and variables

2. Numerics

3. Illustration

4. Relationship

5. Solve